

Taking a Child's Perspective on Number

As adults, we use and understand number in a variety of ways. The connections and differences in these uses can be subtle and even contradictory to young learners. For example, the ordinal number 'third' does not seem to relate to a child who knows he is three years old or that there are three sides to a triangle. Sometimes it is the change in context that is challenging and other times it's the concepts themselves. Confusion may lead to guessing or giving up, and yet we know a strong number sense means the child is starting to make connections between all of these concepts!

Early counting experiences set the stage for disposition towards math, and the goal is to set an enjoyable and successful foundation for all future math inquiry. This activity is designed for adults to see the variety of ways we speak about numbers with young children. If we are able to be explicit and intentional about counting, children will develop a deep understanding of number and confidence in learning more mathematics.

Launch: This activity is meant to explore some of the ways we use numbers.

Contemplate the following concepts of number and counting that young children are learning. Consider the ways in which the concepts may be connected or seemingly contradictory to them. Consider also the subtle differences and what it might look like when a child is confused about a concept.

Counting sequence: 1,2,3,4

Ordinal numbers: The fourth student in line is the last.

Cardinality: The last number in the count sequence represents the total number in that set.

Time concepts: You are four years old. It is four o'clock.

Conservation of number: This is four :: and this is four When I cover the dots and you can't see them that is four also!

Compose and decomposing numbers: 4 is 3 and 1. 3 and 1 is 4.

Numeral and word symbols: :: is 4 and it is also spelled four.

Subitizing: You don't need to count. You can just see it and know how many there are in that set.

1:1 Correspondence: Count each item once and only once, count slowly, and keep track of which item you have counted.

Math in Literature

In your read-aloud books, find concepts of number, size, shape, pattern, or things you could classify. Use post-it notes to write your ideas down on the page. Refer to the CROWD strategies with a math lens:

Completion:

The reader creates an incomplete sentence to prompt the children to come up with response. This is a powerful **to model patterns** in books i.e. predictability. (Ex. Mama called the doctor and the doctor said_____".) Include mathematical language such as extending the pattern, and pointing out what is not a pattern and why. When the book is done, use the idea of pattern to extend the story. You could also write your own pattern book with the children.

Recall:

The reader asks a question designed to help children remember key elements of the story. Think of **mathematical concepts to recall** (Ex. There are 3 monkeys now. How many were there in the beginning of the book?)

Open-ended:

The reader asks a question that requires children to describe part of the story in their own words beyond a yes or no response. Try to think of open-ended questions with **targeted math concepts** (Ex. Isn't it amazing that the peddler had exactly the same number of caps as there are monkeys? How do you think that happened?")

Wh-questions:

The reader asks a question about the story that begins with what, where, who, or why. To focus on **spatial vocabulary ask frequent where questions**. (Ex. Where are the monkeys? Answer: In the tree! Where are the monkeys in relation to the peddler? The monkeys are above the peddler.)

Distancing: The reader helps the children make connections between events that happen in the story to those in their own lives. **Target a math concept such as relative size**. (Ex. Clifford grew bigger and bigger in the story. Can you tell me a time when you saw something get smaller and smaller?)

Outdoor Math Prompts

The SFUSD Early Ed Dept is in the process of painting templates on the blacktops of schools to support outdoor explorations of key early math concepts. The images include a **number line**, a **ten frame**, a **Venn diagram**, a **pattern template**, **three shapes**, and a **T chart**.

The images will be used as a kinesthetic exploration of the concepts of math common core state standards: 1:1 correspondence, cardinality, creating sets, identifying numerals, numbers sense such as more than, less than, the same as, composing and decomposing number, and geometric concepts such as shape, side, corner and angle. There will be positional concepts as well, such as next to, inside, above, in front of and forward. There will be opportunities to explore algebra and classification concepts. The images will also be used for groups of children to build skills in social emotional literacy, physical development and science.

The images are intentionally aligned with the CCSS-Math. We have provided professional development meetings to teachers on how the images can be used to teach specific math concepts.



Outcomes:

Children will experience math in a physical way, which supports children who may have difficulty in learning math in the classroom. Movement and social games will support a variety of learning modalities. The outdoor math movement games will provide an alternative experience of a concept children are beginning to master. Children will master the following math concepts: counting and cardinality, creating sets, identifying numerals, creating patterns, more than, less than, shapes, positional concepts, patterns, algebraic symbols, and classification.

Ball Roll

In this activity, children will manipulate numbers physically. They will playfully find ways to compose and decompose. This activity introduces the idea of number bonds.

Launch: The teacher will use an addition board with a small group of children. Students place small balls or pom poms into each of the paper tubes to create a new quantity in the bottom container.

For example, a student places 2 balls into the tube on the left and 3 balls into the tube on the right. The student then counts the balls in the bottom container to get 5.

During: Students can use the balls in the bottom and split them in a new way, showing that numbers can be decomposed in different ways. This demonstrates that numbers can be grouped together in mathematical ways, depending on the action or function.

Closure: The teacher narrates the actions. "The first time you put 3 and 2 balls together to make 5 in the box. The second time you put 4 and 1 ball together to make 5. Let's show the different ways you made five using the fingers on your hand.

Potential Extension: Teachers can add an addition and equal sign and attach numerals onto the board. For example, the student selects the numeral 2, places it above the left tube, and then puts 2 balls in the left tube. Repeats with another numeral for the right tube. Then finally places the sum of the two above the container. The board now also included the equation $2+3=5$.

CA Preschool Foundations: Number Sense 1.1, 1.2, 1.4, 1.5, 2.0, 2.3, 2.4

Bingo Series

Our Bingo Series is a simple approach to subitizing small numbers and identifying numerals. The key to this activity is to match your students to a particular level. It's possible to choose a different board for different students as they all work together in one small group. The level is based on their understanding of number. One child may be ready to match dots in a different arrangement while another might match dots in an identical arrangement. The board you give to a child sets the level of difficulty. Another example is children who are ready to match numeral to dot arrangement versus children who match numeral to numeral. The first few times you play, everyone should have the same level of board. As children become familiar with the play, you can insert different level of boards for different children.

Launch: In a small group, demonstrate how the bingo board works. Explain that you want to match the number of dots or number on top of the die to the same on the bingo board. Each board increases in difficulty. Arrange your small groups based on the level of each student. If a student needs support, give that student more time to play the same bingo game multiple times throughout the week. Other students may be ready to move up to the next board right away.

During: In the first board, the primary focus is to identify numerals 1-5.

The second board focuses on subitizing numbers 1-3 with matching arrangements. So the essential step in both these boards is to ask the student what number they rolled. Make notes of correct and incorrect answers for each student.

The third board emphasizes matching numerals 1-3 to a set of dots so the emphasis is on subitizing the dots correctly and identifying numerals.

The fourth board focuses on matching sets of dots to numerals 1-5. In this level subitizing, counting, and identifying numerals are all combined.

The fifth board focuses on matching a set of dots 1-5 to another set of dots 1-5 arranged differently. In this level subitizing, counting, and identifying numerals are all combined.

Potential Misconceptions: Students might be confused with quantity when objects are arranged differently. Students may subitize incorrectly. In this case, use a smaller number of objects, count, ask again and count again. Also, provide more subitizing practice throughout the day. Also, students will practice identifying numerals throughout the day, targeting numerals that children are still learning based on errors they made during Bingo.

CA Preschool Learning Foundations: Number Sense 1.1, 1.2, 1.3, 1.4, 1.5

Fistful of Counters

This activity is an introduction to the 10-frame and provides a visual exploration of number. It calls for math discourse, and is an opportunity to attend to two of the standards for math practice: MP3 Construct viable arguments and critique the reasoning of others; and MP5: Use appropriate tools strategically.

Launch: Fill two bowls with different sized counters. One bowl should be counters that are identical and graspable such as slinking cubes. The other bowl should be counting objects that are larger, also identical and graspable, such as small blocks or corks.

Each student will need a ten frame to record their work.

Students take turns using one hand to grasp objects from one bowl. They place the objects on the ten-frame, one per box.

After they place the objects, they count them one at a time, slowly touching each object as they say the number out loud.

During: As students take turns, write down their names, the number of bears they grasped with one hand, and note any errors and comments the child made. This will serve as a quick assessment of verbal counting skills and 1:1 correspondence.

To assess and practice cardinality, you should always ask, "How many?" after a child counts. If they do not answer, count again with the child and model the answer; "Five. There are five bears." Invite the children to try it a second or third time, keeping track of each result.

Extension/ Closure: Invite children to write or draw the number of bears they grasped using the ten frame or a blank piece of paper. Discuss how they came up with the answers. Who grasped the same number of bears? Why did I grasp more? What did you notice about the number of blocks you grasped? Why?

After using the ten frame, invite the children to find the numeral that matches the numbers of bears they grasped and invite them to write that number.

CA Preschool Learning Foundations: Number Sense 1.1, 1.2, 1.3, 1.4, 1.5

Number Hunt

In this activity, children first search for and then identify numbers. They can be written as numerals or number of dots on a card. Next, they match that number to a basket with a number. This gives children an active experience of sorting and subitizing.

Description of Lesson: Number Hunt

Hide numerals 1-5 throughout the classroom, each teacher has two baskets labeled with numerals or dots. The children find numeral or dot cards, find the corresponding basket and then search for more numeral or dot cards.

Notes:

This is an active lesson with all students and all teachers participating. Hide the numerals beforehand. Introduce the activity in morning circle time and then give the children time to search for the numerals. There should be enough hidden numerals so that each child can find at least 3 in the classroom.

The first level of difficulty is matching numeral to numeral. More difficult is matching numerals to dots typically arranged, such as seen on dice. Most difficult is matching dots arranged typically with dots arranged in a non-typical way.

Materials:

Numerals- could be written on card stock, magnetic numbers or another commercially purchased material, also Baskets or small bowls.

Extension Options:

Children can make their own numeral or dot cards and label basket to hide for other students.

Create a few hidden dot cards that do not have corresponding baskets. Ask the children what you can do with these numbers (listen for responses such as label a new basket, add two numbers together to equal one of the basket numbers, tear the dot picture and add them to two different baskets etc.) For example, :: Could be torn as : and :. And added to the 2 and 3 basket.

CA Preschool Learning Foundations: Number Sense 1.2, 1.3, 1.4, 1.5

Number Stories

The most effective stories relate to a classroom project. For example, if you were doing a bird study, it would make sense for ducklings to choose between a lake and a nest. Cubes can represent any animal or person.

Launch: In small group, model an animal story beginning with an emphasis on a particular number 3-5. Introduce two locations on a template (tree and house) with that particular number and have that many animals to act out the story with animal counters. The story will have an element of leaving and returning (decomposing and composing).

During: The teacher will model one or two stories. Then, in small groups, ask children to tell stories with animals using the template. One child will tell the story and the other children will use their animal counters and templates to act out the story. The children can also work with a partner.

Teacher: "How many dogs are there in your story?"

Child: "5"

Teacher: "Let's start with the animals all together on this plate. Where do you want your dogs to go, on the tree or in the house?"

Child: "Lake"

Teacher: "How many animals are going to the lake?"

Child: "4"

Teacher: "Okay, so put 4 animals in the lake. How many animals are left to go to the house?"

Child: "1"

Teacher: "So how many animals in the lake? How many in the house?"

Teacher: "Now the sky is getting dark and the air is getting cold. The animals want to go home and be together on the plate again. Can you bring them back home so they are all together?"

Teacher: "How many animals are there all together?"

Child: "5"

Extension/Closure: Students may compare which area has more animals. The teacher can model other hand drawn templates such as rocket ships, birthday parties, and a playground. Then children can draw their own templates, share their stories with friends and family.

One Color Post-It Math

In this activity, children create a set of a number with post-its Notes. In doing so, they are creating an example of a number visually to show other students.

Launch: The teacher explains the Post-It Note rules. The teacher has cut the size of the paper to decrease or increase the number of Post-Its Notes to fit.

Teacher: You can use as many Post-It Notes as you want! Here are the two rules: 1. A Post It note cannot touch another Post-It note. 2. The Post It notes must be completely on the paper. It cannot go off the sides of the paper!

During: The child then arranges Post-Its notes independently.

Students are then asked to describe the work on their paper. The teacher may write down the numeral or show it in a ten frame or represent the number with the same number of dots.

Closure: The teacher may ask these follow up questions to determine related number sense concepts.

“If we take away one note how many do we have?”

“Which color has more/less? How could we make them the same or equal?”

Potential Extension: Ask student to show that number in a different way (numeral, ten frame, dot arrangement) The teacher can display examples of sets of six together on the wall, showing different ways to represent six. Children can create sets with other objects at home, such as bottle caps and pennies, and describe them to their family members.

CA Preschool Learning Foundations:

Number Sense 1.1, 1.2, 1.3, 1.4, 1.5, 2.0, 2.1, 2.2, 2.3, 2.4

Two Color Post-It Math (Composing)

In this activity, children create a set of numbers with post-its notes, choosing the number set and choosing the number of each post-it note color. In doing so, they are creating an example of a number family to show other students.

Launch: The teacher explains the Post-It Note rules. Students select Post It Notes of two different colors and are given a piece of plain 8 1/2 x 11 piece of paper. You can cut the size of the paper or use legal size to decrease or increase the number of Post-It Notes to fit.

Teacher: You can use as many Post-It Notes as you want and you can choose which colors you want to use too! (There are only two colors presented) Here are the two rules: 1. A Post It note cannot touch another Post-It note. 2. The Post It notes must be completely on the paper. It cannot go off the sides of the paper!

During: The child then arranges Post-Its notes independently, choosing number and color.

Students are then asked to describe the work on their paper. An answer could resemble, "I have 3 blue and 1 yellow." The teacher may write down those numerals. She may then ask, "How many Post-Its notes do you have all together?" She may write that numeral down as well.

Closure: The teacher may ask these follow up questions to determine related number sense concepts.

"Which color has more notes, the blue or the yellow?" "How many more blues notes do you have?" "If we take away the yellow notes how many do we have?"

Potential Extension: Ask students to write equations. Add a third color of Post-It Notes. The teacher can display examples of sets of six together on the wall, showing different ways to compose and decompose those numbers. Children can create sets with other objects at home, such as bottle caps and pennies, and describe them to their family members.

CA Preschool Learning Foundations: Number Sense 1.1, 1.2, 1.3, 1.4, 1.5, 2.0, 2.1, 2.2, 2.3, 2.4

Early Childhood Math Trajectories

The Early Childhood Math Trajectories outline the objectives for each sub-category of preschool mathematics. The objectives have been placed in sequential order based on Julie Sarama and Doug Clements' compilation of research on children's development of mathematical concepts.

Note:

- Concepts from California Preschool Learning Foundations will be referred to as PLF and
- Concepts from Doug Clements' "Learning and Teaching Early Math" will be referred to as DC
- Concepts from Common Core will be referred to as CC

Overarching Concepts

These standards should serve as the overarching concepts for math instruction. It is essential that teachers hold these standards in mind during the planning and implementation of all lessons and tasks.

Focus Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Reasoning

1. Begin to apply simple mathematical strategies to solve problems in their environment.
2. Identify and apply a variety of mathematical strategies to solve problems in their environment.

Number Sense (Counting)

1. Identify, without counting, the number of objects in a set of up to 3. (PLF, DC)
2. Verbally counts with separate words, perhaps in random order, up to 5.
3. Create a set of objects up to 3. (DC)
4. Recite numbers in order to 10 with increasing accuracy. (PLF, DC)
5. Identify, without counting, the number of objects in a set of up to 4. (PLF, DC)
6. Create a set of objects up to 4. (DC)
7. Count up to 5 objects, using 1-to-1 correspondence (once object for each number word) with increasing accuracy. (PLF, CC, DC)

8. Begin to recognize and name a few written numerals. (CC, PLF, DC)
9. Identify number just after or just before another number but only by counting up from 1 (e.g. What comes after 4? 1, 2, 3, 4...5). Use the number name of the last object counted to answer the question, "How many...?" (PLF)(DC)
10. Recite numbers in order to 20 with increasing accuracy. (PLF, DC)
11. Create a set of objects up to 5. (DC)
12. Count up to 10 objects, using 1-to-1 correspondence (one object for each number word) with increasing accuracy. (PLF, CC, DC)
13. Understand, when counting, that the number name of the last object counted represents the total number of objects in the group: Cardinality. (PLF, CC, DC)
14. Recognize, know the name of, and write numerals 1-10. (PLF, DC, CC)
15. Count forward beginning from a given number of a known sequence (CC)
16. Count backwards from 10-1 verbally or when removing objects from a group (DC)

Classification and Patterning

Classification:

1. Identify characteristic of objects.
2. Identify when objects are the same and/or different.
3. Sort and classify objects by one attribute into two or more groups, with increasing accuracy. (CC, PLF)
4. Sort and classify objects by one or more attribute into two or more groups, with increasing accuracy. (CC,PLF)

Patterning:

(Note: do not begin until children have mastered level 3 of classification).

1. Begin to identify or recognize a simple repeating pattern that has units. (PLF, DC)
2. Fills in missing element of an AB pattern (DC)
3. Attempt to create a simple repeating pattern or participate in making one. (PLF)
4. Duplicate an AB pattern. (PLF, DC)
5. Extend AB pattern. (PLF, DC)
6. Duplicate an ABB and/or ABC pattern (PLF, DC)
7. Extend ABB or ABC pattern (PLF, DC)
8. Identifies the smallest set of a pattern. (DC)

Geometry

1. Identify characteristic of objects.
2. Identify when objects are the same and/or different.
3. Matches familiar shapes (circle, square, triangle) with same size and orientation. (DC)
4. Matches familiar shapes (circle, square, triangle) with different sizes and same orientation. (CC, DC)

5. Matches familiar shapes (circle, square, triangle) with different orientations. (DC)
6. Identify by name simple two-dimensional shapes, such as a circle, square, and triangle. (DC)
7. Recognizes shapes within the environment (CC)
8. Use individual shapes to represent different elements of a picture or design (overlay). (PLF)
9. Combine different shapes to create a picture or design. (PLF)
10. Matches a wider variety of shapes (non-typical triangle, rectangles, ovals, shape clusters, etc.) (DC)
11. Identify by name, describe, and construct a variety of different shapes, including variations of a circle, triangle, rectangle, square, and other shapes. (PLF, DC)
12. Uses manipulatives representing parts of shapes such as sides to make a shape that looks like a goal shape, e.g. using sticks to create a triangle. (DC)
13. Identify positions of objects and people in space, including above/below, up/down, inside/outside, beside/between, and in front/behind. (PLF, CC)
14. Identify sides as geometric objects. (DC)
15. Identifies angles/corners as geometric objects. (DC)
16. Recognizes most familiar shapes and typical examples of other shapes such as hexagon, rhombus, trapezoid, and 3-D shapes. (DC, CC)

Mathematical Concepts (Number Sense 2)

Comparing

1. Compares sets that are "quite" different in size (e.g. one set is at least twice the other set.) (DC, PLF:NS 2.1)
2. Compares sets of 1-4 items. The items must be the same (e.g. 3 dots and 3 dots). (DC, PLF:NS 2.1)
3. Matches small equal sets of different objects (e.g. 3 dots and 3 shells). (DC, PLF:NS 2.1)
4. Compares groups of 1-6 by matching (e.g. pairing objects such as children to chairs). (DC, PLF:NS 2.1)
5. Accurately compares via counting when objects are similar in size and the sets are small. (DC, PLF:NS 2.1)

Addition/Subtraction

1. Understand that adding to (or taking away) one or more objects from a group will increase (or decrease) the number of objects in the group. (PLF, CC)
2. Understand that putting two groups of objects together will make a bigger group. (PLF)
3. Understand that putting two groups of objects together will make a bigger group and that a group of objects can be taken apart into smaller groups. (PLF)
4. Solve simple additions and subtraction problems nonverbally (and often verbally) with a very small number of objects (sum up to 4 or 5). (PLF, DC)
5. Solve simple addition and subtraction problems with a small number of objects (sums up to 10), usually by counting. (PLF)
6. Understand that adding one or taking away one changes the number in a small group of objects by exactly one. (PLF, DC, 6 years old)

Measurement

1. Identify characteristic of objects.
2. Identify when objects are the same and/or different.
3. Identifies length/distance as an attribute. May understand length as an absolute descriptor (e.g. All adults are tall). (DC)
4. Demonstrate awareness that objects can be compared by length, weight, or capacity, by noting gross differences, using words such as bigger, longer, heavier, or taller, or by placing objects side by side to compare length. (PLF)
5. Compare two objects by length, weight, or capacity directly: putting objects side by side. (PLF, CC, DC)
6. Order 3 objects by size. (PLF)
7. Order 4 or more objects by size. (PLF)
8. Compare two objects by length, weight, or capacity indirectly: using a third object. (PLF, CC)
9. Measure using multiple duplicates of same-size concrete units laid end to end. (PLF, DC: 6 years old)